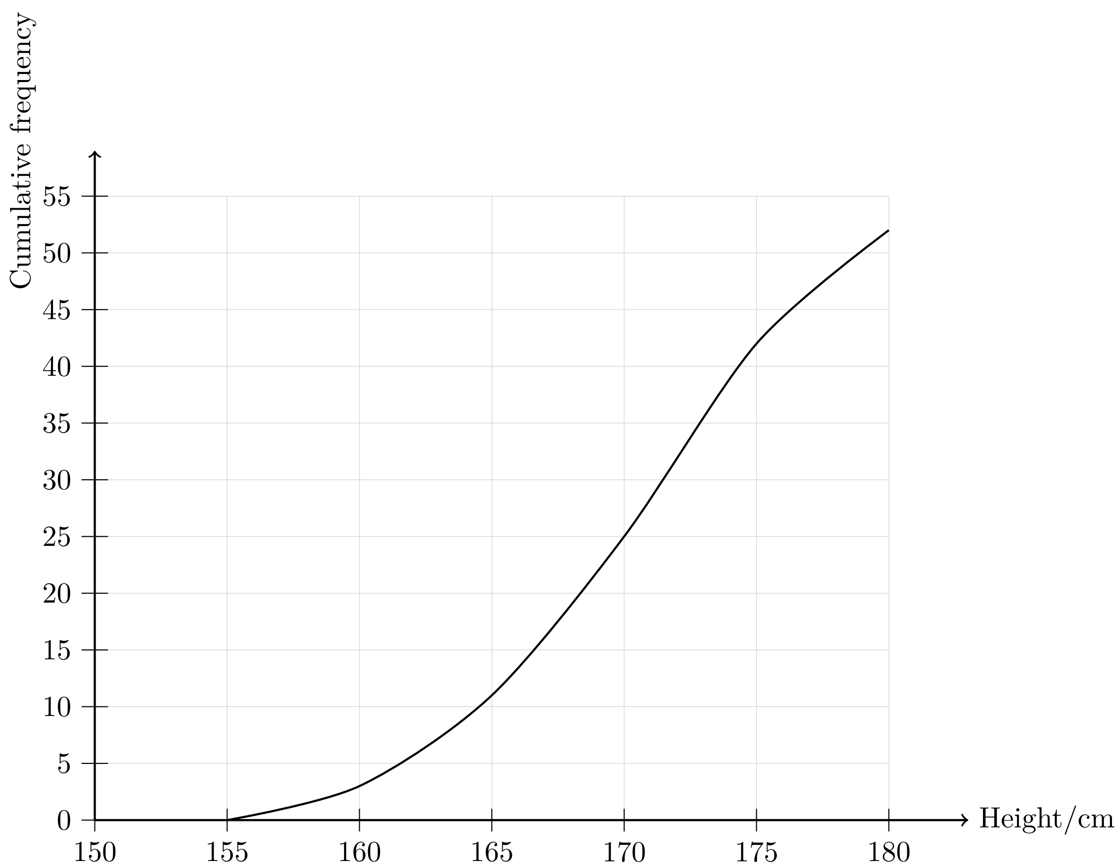
Mathematics: analysis and approaches Higher level
Paper 2

Section A

1. [Maximum mark: 7]

A group of 52 students have their heights measured in cm. The cumulative frequency graph for the heights of the students is shown in the following diagram.



(a) Use the graph to find an estimate of the median height. [2]

The heights of the students are shown in the following frequency table where $p, q \in \mathbb{N}$.

Height (h cm)	Frequency
$155 \leq h < 160$	3
$160 \leq h < 165$	8
$165 \leq h < 170$	14
$170 \leq h < 175$	p
$175 \leq h < 180$	q

(b) (i) Find the value of p and the value of q.

(ii) Use the frequency table to find an estimate of the mean height, giving your answer to four significant figures. [5]

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2. [Maximum mark: 4]

Consider a function f which has a first derivative given by $f'(x) = x^2 - x - 1$, where $x \in \mathbb{R}$.

Find the range of values of x for which f is decreasing. [4]

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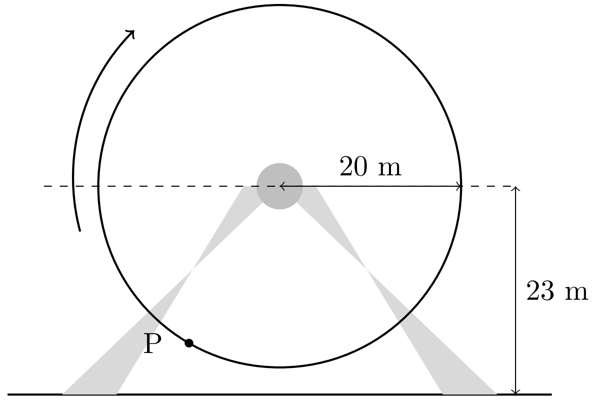
3. [Maximum mark: 6]

A Ferris wheel with radius 20 metres rotates at a constant speed. The centre of the wheel is 23 metres above the ground, as shown in the following diagram. P is a point on the wheel.



The wheel starts moving with P at the lowest point and completes one revolution in 4 minutes.

The height, h metres, of P above the ground after t minutes is given by $h = a - b\cos(ct)^\circ$, where $a, b, c \in \mathbb{Z}^+$.

(a) Find the value of a, the value of b and the value of c. [4]

The height can also be given as $h(t) = a - b\sin(c(t+d))^\circ$, where $d \in \mathbb{Z}^+$.

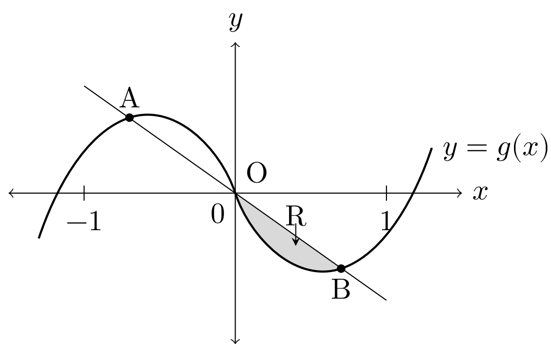
(b) Find the smallest positive value of d. [2]

4. [Maximum mark: 7]

Consider the function g defined as $g(x) = x^3 - 3x + 2\sin x$, where $x \in \mathbb{R}$.

The graph of g has a local maximum at point A and a local minimum at point B. The line (AB) passes through the origin O at (0,0).

The region R is enclosed by the graph of g and the line segment [OB], as shown on the following diagram.



(a) Find the coordinates of point A and the coordinates of point B. [2]

(b) Find the area of R. [5]

5. [Maximum mark: 7]

A large group of adults have their heights measured. The heights of the adults are normally distributed with mean μ cm and standard deviation σ cm.

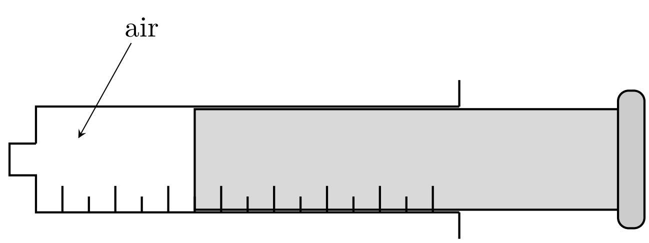
The probability that one of the adults chosen at random has a height greater than 171 cm is 0.4.

If one of the adults with a height greater than 171 cm is chosen at random, the probability that their height is greater than 175 cm is 0.35.

Find the value of μ and the value of σ. [7]

6. [Maximum mark: 4]

The air inside an airtight syringe is compressed so that its volume, $V\,\text{cm}^3$, decreases.



The air pressure, P Newtons per cm^2, within the syringe is inversely proportional to the volume so that $PV = 135$.

At a given time, the volume of air is 20 cm^3 and is decreasing at a rate of 8 cm^3 per minute.

Find the rate of increase in the air pressure at this time. [4]

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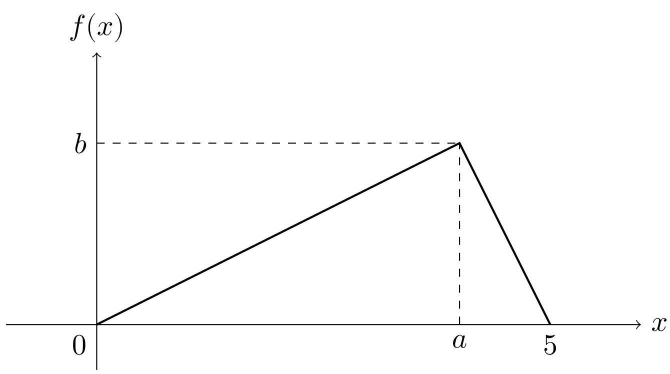
7. [Maximum mark: 6]

The continuous random variable X has a probability distribution f defined as

$$f(x) = \begin{cases} \frac{b}{a}x, & 0 \leq x \leq a \\ \frac{b}{a-5}(x-5), & a < x \leq 5 \end{cases}$$

where $0 < a < 5$ and $b \in \mathbb{Q}^+$.

The graph of f is shown in the following diagram.



(a) Show that $b = 0.4$. [2]

It is given that $\mathrm{E}(X) = 3.2$.

(b) Find the value of a. [4]

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8. [Maximum mark: 8]

Consider the equation $z^3 = c$, where $z \in \mathbb{C}$ and c is a positive constant.

One of the solutions of the equation is $z = -2 + 2i\sqrt{3}$.

(a) Find the value of c. [2]

(b) Write down the other two solutions of $z^3 = c$ giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$. [2]

(c) Find the solutions of $(iz)^3 = c$ giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$. [2]

The solutions of $(iz)^3 = c$ can be represented on an Argand diagram by three points A, B and C.

(d) Find the area of the triangle ABC. [2]

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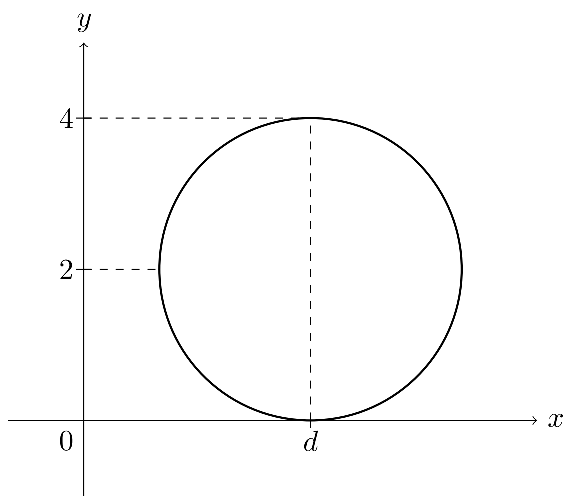
9. [Maximum mark: 8]

Consider the curve $(x-d)^2+(y-2)^2=4$ where $x, y \in \mathbb{R}$ and $d>2$, shown in the following diagram.



The region enclosed by the curve is rotated through 2π radians around the y-axis to form a solid, S.

(a) Show that the volume of S is given by $4\pi d \int_0^4 \sqrt{4-(y-2)^2}\,dy$. [6]

It is given that the volume of S is $50\pi^2$ cm^3.

(b) Find the value of d. [2]

Section B

10. [Maximum mark: 13]

At the start of 2026, Lavinia receives a gift of \$15000. She wants to buy a boat which costs \$28000 so she decides to invest her money.

On 1 January 2026 Lavinia invests her money in a bank account which pays interest at a nominal annual rate of 4.8%, compounded quarterly. The interest is paid into her account on the last day of each quarter. She makes no further deposits to, or withdrawals from, the account.

(a) Find the amount of money Lavinia will have in her bank account on 1 January 2031. Give your answer correct to the nearest dollar. [3]

(b) Show that Lavinia will first have more than \$28000 in her bank account during the year 2039. [3]

The cost of the boat at the start of 2026 was \$28000, and it depreciates at a constant annual rate of $r\%$. After one year the cost of the boat is \$26222.

(c) Find the value of r. [2]

Due to this depreciation, Lavinia will be able to buy the boat before 2039. She will buy the boat as soon as she has enough money in her bank account to pay for it.

(d) Determine the year during which Lavinia will buy the boat. [5]

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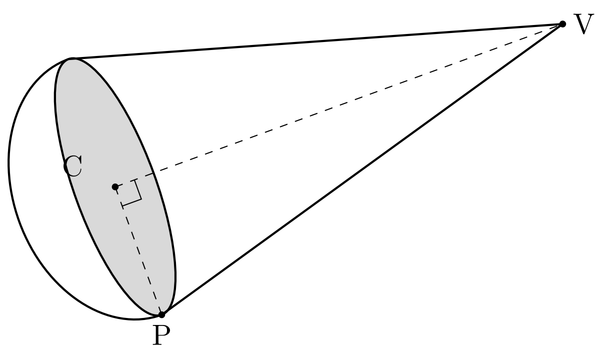
11. [Maximum mark: 19]

A child's toy is made from a solid right cone and a solid hemisphere of equal radius. The circular faces of each solid are joined to form a single circle C. The point P lies on the circumference of C, and the point V is the vertex of the cone as shown in the following diagram.



In this question, all units are given in cm.

The toy is positioned so that the circle C lies completely within the plane defined by $x + 2y - z = 5$.

The point P has coordinates $(-3, 6, 4)$ and the point V has coordinates $(4, 16, -5)$.

(a) Find a vector equation for the line passing through V and the centre of C. [3]

(b) Find the coordinates of the centre of C. [3]

(c) (i) Show that the radius of C is $\sqrt{14}$ cm.
(ii) Find the total surface area of the toy. [6]

The toy is rotated so that the position of the vertex is fixed at V and the image P' of the point P has coordinates $(-9, 11, 1)$.

(d) Find angle $P\hat{V}P'$ giving your answer in degrees. [3]

(e) Find an equation of the plane containing the points P, V and P', giving your answer in the form $ax + by + cz = d$, where $a, b, c, d \in \mathbb{Z}$. [4]

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12. [Maximum mark: 21]

The function f is defined as $f(x) = \frac{x^2+ax+b}{x+1}$, where $x \neq -1$ and $a, b \in \mathbb{Z}$.

First consider the case where $\lim_{x\to-1} f(x)$ is finite.

(a) (i) Show that $b = a - 1$.
(ii) Find $\lim_{x\to-1} f(x)$ in terms of a. [4]

Now consider the case where the graph of f has a vertical asymptote, an oblique asymptote with equation $y = x - 4$, and at least one point of zero gradient.

(b) Write down the equation of the vertical asymptote. [1]

(c) Show that $a = -3$. [3]

(d) Show that $b \geq -3$. [6]

(e) In the case where $a = -3$ and $b = 1$

(i) sketch the graph of $y = f(|x|)$, showing any asymptotes with their equations, the value of the intercept with the y-axis, and the coordinates of any local maximum or minimum points;
(ii) solve $f(|x|) < f(x)$. [7]